Further Kinematics - Answers

June 2017 Mathematics Advanced Paper 1: Mechanics 1

1.

Question Number	Scheme	Marks
7(a)	$\tan\theta=\frac{2}{9}$ $\theta=12.5^\circ$ bearing 103°	M1 A1 A1 (3)
(b) **(i)** **(ii)**	$\mathbf{p}=(9\mathbf{i}+10\mathbf{j})+t(9\mathbf{i}-2\mathbf{j})$ $\mathbf{q}=(\mathbf{i}+4\mathbf{j})+t(4\mathbf{i}+8\mathbf{j})$	M1 A1 A1 (3)
(c)	$\overrightarrow{QP}=(8+5t)\mathbf{i}+(6-10t)\mathbf{j}$	M1 A1 (2)
(d)	$D^2=(8+5t)^2+(6-10t)^2$ $=125t^2-40t+100$ $100=125t^2-40t+100$ $0=5t(25t-8)$ $t=0$ or 0.32	M1 A1 M1 M1 A1 A1 (6) **14**
	Notes	
7(a)	M1 for $\tan\theta=\pm\frac{2}{9}$ or $\pm\frac{9}{2}$ or use $\sin\theta$ or $\cos\theta$	
	First A1 for $\theta=\pm13^\circ$ or $\pm77^\circ$ or $\pm12.5^\circ$ or $\pm77.5^\circ$ or better	
	Second A1 for 103°	
7(b)	M1 for clear attempt at $\mathbf{p}=(9\mathbf{i}+10\mathbf{j})+t(9\mathbf{i}-2\mathbf{j})$ or $\mathbf{q}=(\mathbf{i}+4\mathbf{j})+t(4\mathbf{i}+8\mathbf{j})$ (Allow slips but must be a '+' sign and $\mathbf{r}+t\mathbf{v}$)	
(i)	First A1 for $\mathbf{p}=(9\mathbf{i}+10\mathbf{j})+t(9\mathbf{i}-2\mathbf{j})$oe	
(ii)	Second A1 for $\mathbf{q}=(\mathbf{i}+4\mathbf{j})+t(4\mathbf{i}+8\mathbf{j})$oe	
7(c)	M1 for $\mathbf{p}-\mathbf{q}$ or $\mathbf{q}-\mathbf{p}$ with their $\mathbf{p}$ and $\mathbf{q}$ substituted A1 for correct answer $\overrightarrow{QP}=(8+5t)\mathbf{i}+(6-10t)\mathbf{j}$ (don't need $\overrightarrow{QP}$ but on R.H.S must be **identical** coefficients of $\mathbf{i}$ and $\mathbf{j}$ but allow column vectors)	
7(d)	First M1 for attempt to find QP or QP^2 in terms of t *only*, using correct formula First A1 for a correct expression (with or without $\sqrt{}$) $125t^2-40t+100$ Second M1 for $\sqrt{}$ (3 term quadratic) = 10 or (3 term quadratic) =100. Third M1 for quadratic expression = 0 and attempt to solve (e.g. factorising or using formula) Second A1 for $t=0$ (if they divide by t and lose this value but get 0.32, M1A0A1) Third A1 for $t=0.32$ oe	

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2.

Question Number	Scheme	Marks
1(a)	$\tan\theta=\frac{5}{20}$ $\theta=14.036..^{0}$ $\theta=104^{0}$ nearest degree	M1 A1 A1 (3)
(b)	$\mathbf{p}=400\mathbf{i}+t(15\mathbf{i}+20\mathbf{j})$ $\mathbf{q}=800\mathbf{j}+t(20\mathbf{i}-5\mathbf{j})$	M1 A1 A1 (3)
(c)	Equate their **j** components: $20t(\mathbf{j})=(800-5t)(\mathbf{j})$ $t=32$ $\mathbf{s}=800\mathbf{j}+32(20\mathbf{i}-5\mathbf{j})$ $=640\mathbf{i}+640\mathbf{j}$	M1 A1 M1 A1 (4) **10**

	Notes
1(a)	**Allow column vectors throughout** M1 for $\tan\theta=\pm\frac{5}{20}$ or $\pm\frac{20}{5}$ (or any other complete method) First A1 for $\pm14.04^{o}$ or $\pm75.96^{o}$ Second A1 for 104^{o}
1(b) **(i)** **(ii)**	M1 for clear attempt at either **p** or **q** (allow slip but t must be attached to the velocity vector and position vector and velocity vector must be paired up correctly) First A1 $400\mathbf{i}+t(15\mathbf{i}+20\mathbf{j})$ "**p** =" not needed but must be clear it's P Second A1 $800\mathbf{j}+t(20\mathbf{i}-5\mathbf{j})$ "**q** =" not needed but must be clear it's Q
1(c)	First M1 for equating their **j** components; allow **j**'s on both sides First A1 for $t=32$ Second M1 independent for substituting their t value into their **q** from (b) Second A1 for $640\mathbf{i}+640\mathbf{j}$

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3.

Question Number	Scheme	Marks
6(a)	$\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j})$ m	B1 (1)
(b)	$3.4 = 2T - 3$ or $-12 = 4 - 5T$ $T = 3.2$	M1 A1 A1 (3)
(c)	$\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j}) + t(2\mathbf{i} - 5\mathbf{j})$ $\mathbf{v} = (2\mathbf{i} - 5\mathbf{j})$ speed $= \sqrt{\left(2^2 + (-5)^2\right)} = \sqrt{29} = 5.4\text{m s}^{-1}$ or better	M1 A1 M1 A1 (4) **8**
Alt (c)	$\lvert\mathbf{s}\rvert = \sqrt{6.4^2 + (-16)^2} = 17.23...$ $\therefore$ speed $= \frac{17.23}{3.2} = 5.4$ or better	M1 A1 M1 A1 (4)
	Notes	
6(a)	**Allow column vectors throughout.** B1 for $(-3\mathbf{i} + 4\mathbf{j})$ (m)	
(b)	M1 for a clear attempt at either 3.4 (**i**)= $(2T - 3)$ (**i**) or -12(**j**) = $(4 - 5T)$ (**j**) First A1 for a correct equation (either) without **i**'s and **j**'s A1 for 3.2 oe **N.B.** $T = \frac{6.4\mathbf{i} - 16\mathbf{j}}{2\mathbf{i} - 5\mathbf{j}} = 3.2$ scores M1A1A1 BUT if RHS is not a single number, then M0. Also, if they get 3.2 and another value and don't clearly choose 3.2 then A0	

(c)	First M1 for a complete method for finding **v** e.g. $\mathbf{r}=(-3\mathbf{i}+4\mathbf{j})+t(2\mathbf{i}-5\mathbf{j})$ so $\mathbf{v}=2\mathbf{i}-5\mathbf{j}$ OR: $\mathbf{v}=\frac{(3.4\mathbf{i}-12\mathbf{j})-(-3\mathbf{i}+4\mathbf{j})}{\text{their } T}$ OR: $\mathbf{v}=\frac{d\mathbf{r}}{dt}=2\mathbf{i}-5\mathbf{j}$ First A1 for $2\mathbf{i}-5\mathbf{j}$; M1A1 can be awarded for $2\mathbf{i}-5\mathbf{j}$ only. Second M1 for attempt to find magnitude of their **v**, i.e. $\sqrt{2^2+(-5)^2}$ Second A1 for $\sqrt{29}$ or 5.4 or better **OR** First M1 for attempt to find distance travelled: $d=\sqrt{(-3-3.4)^2+(4--12)^2}$ First A1 if correct Second M1 for their d / their T Second A1 for $\sqrt{29}$ or 5.4 or better	

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4.

Question Number	Scheme	Marks
5a	$\mathbf{F}=m\boldsymbol{a}$: $3\mathbf{i}-2\mathbf{j}=0.5\boldsymbol{a}$ $\boldsymbol{a}=6\mathbf{i}-4\mathbf{j}$ $\lvert\boldsymbol{a}\rvert=\sqrt{6^2+(-4)^2}=2\sqrt{13}\,(\text{m s}^{-2})$ **	M1 A1 M1A1 (4)
b	$\boldsymbol{v}=\boldsymbol{u}+\boldsymbol{a}t$: $\boldsymbol{v}=(\mathbf{i}+3\mathbf{j})+2(6\mathbf{i}-4\mathbf{j})$ $=13\mathbf{i}-5\mathbf{j}$ m s^{-1}	M1A1 ft A1 (3)
c	Distance $=2\lvert\mathbf{v}\rvert=2\sqrt{4+1}=2\sqrt{5}=4.47\,(\text{m})$	M1A1 (2)
d	When $t=3.5$, velocity of P is $(\mathbf{i}+3\mathbf{j})+3.5(6\mathbf{i}-4\mathbf{j})=22\mathbf{i}-11\mathbf{j}$ Given conclusion reached correctly. E.g. $22\mathbf{i}-11\mathbf{j}=11(2\mathbf{i}-\mathbf{j})$	M1A1 ft A1 (3)
		[12]

Notes for Question 5

Question 5(a)

Either:
First M1 for use of $\mathbf{F} = m\,\mathbf{a}$
First A1 for $\mathbf{a} = 6\mathbf{i} - 4\mathbf{j}$
Second M1 for $a = \sqrt{(6^2 + (-4)^2)}$ (Allow $\sqrt{(6^2 + 4^2)}$)
Second A1 for $a = 2\sqrt{13}$ (ms^{-2}) **Given answer**

Or:
First M1 for $F = \sqrt{(3^2 + (-2)^2)}$ (Allow $\sqrt{(3^2 + 2^2)}$)
First A1 $F = \sqrt{13}$
Second M1 for $\sqrt{13} = 0.5\,a$
Second A1 for $a = 2\sqrt{13}$ (ms^{-2}) **Given answer**

Question 5(b)
M1 for **(i + 3j)** + (2 x their **a**)
First A1 **ft** for a correct expression
Second A1 for $13\mathbf{i} - 5\mathbf{j}$; isw if they go on to find the speed

Question 5(c)
M1 for $2\sqrt{(2^2 + (-1)^2)}$ or $\sqrt{(4^2 + (-2)^2)}$
A1 for $2\sqrt{5}$ or $\sqrt{20}$ or 4.5 or 4.47 or better

Question 5(d)
M1 for **(i + 3j)** + (3.5 x their **a**) ,or possibly, their (b) + (1.5 x their **a**)
First A1 ft for a correct expression *of form* $a\mathbf{i} + b\mathbf{j}$
Second A1 for given conclusion reached correctly *e.g.* $22\mathbf{i} - 11\mathbf{j} = 11(2\mathbf{i} - \mathbf{j})$ oe **Given answer**

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5.

Question Number	Scheme	Marks
7.		
(a)	$t = 0$ gives $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$	B1
	speed $= \sqrt{1^2 + (-3)^2}$	M1
	$= \sqrt{10} = 3.2$ or better	A1
		(3)
(b)	$t = 2$ gives $\mathbf{v} = (-3\mathbf{i} + 3\mathbf{j})$	M1
	Bearing is 315°	A1
		(2)
(c)(i)	$1 - 2t = 0 \Rightarrow t = 0.5$	M1 A1
(ii)	$-(3t - 3) = -3(1 - 2t)$	M1 A1
	Solving for t	**DM**1
	t = 2/3, 0.67 or better	A1
		(6)
		[11]

Notes for Question 7		
Q7(a)	B1 for **i** – 3**j**. M1 for √ (sum of squares of cpt.s) A1 for √10, 3.2 or better	
Q7(b)	M1 for clear attempt to sub $t = 2$ into given expression. A1 for 315.	
Q7(c)	**(i)** First M1 for $1 - 2t = 0$. First A1 for $t = 0.5$. N.B. If they offer two solutions, by equating both the **i** and **j** components to zero, give M0. **(ii)** First M1 for $\frac{1-2t}{3t-3} = \pm(\frac{-1}{-3})$ o.e. (Must be an equation in t *only*) First A1 for a correct equation (the + sign) Second M1, dependent on first M1, for solving for t. Second A1 for 2/3, 0.67 or better.	

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6.

6.(a)	$\frac{(\mathbf{i}-4\mathbf{j})-(4\mathbf{i}-8\mathbf{j})}{0.5};(\pm 6\mathbf{i} \pm 8\mathbf{j})$ $\sqrt{(\pm 6)^2+(\pm 8)^2}=10$	M1 A1 M1 A1 (4)
(b)	$\mathbf{r}=(4\mathbf{i}-8\mathbf{j})+t(-6\mathbf{i}+8\mathbf{j})$ $=(4\mathbf{i}-8\mathbf{j})-6t\mathbf{i}+8t\mathbf{j}$ $=(4-6t)\mathbf{i}+(8t-8)\mathbf{j}$ *	M1 A1 (2)
(c)	At 10 am, $\mathbf{r}=-2\mathbf{i}$ At 10.30 am, $\mathbf{r}=-5\mathbf{i}+4\mathbf{j}$ $\mathbf{l}=k\mathbf{i},\ k<-2$ $k=-5-4=-9$ $\mathbf{l}=-9\mathbf{i}$	M1 A1 A1 DM1 A1 (5) **11**

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7.

Question Number	Scheme	Marks
6.	(a) $\arctan\frac{7.5}{12} = 32°$	M1 A1
	Bearing is 302 (allow more accuracy)	A1 **(3)**
	(b) $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j} + t(-12\mathbf{i} + 7.5\mathbf{j})$	M1 A1 **(2)**
	(c) $t = 3$, $\mathbf{s} = 4\mathbf{i} + 16.5\mathbf{j}$	M1
	$\mathbf{s} - \mathbf{b} = -3\mathbf{i} + 4\mathbf{j}$	M1
	$SB = \sqrt{\left((-3)^2 + 4^2\right)} = 5 \text{ (km)}$	DM1 A1 **(4)**
	(d) Equating **i** components	
	$40 - 12t = 7$ **or** $-33 + 12t = 0$	M1
	$t = 2\frac{3}{4}$	A1
	When $t = 2\frac{3}{4}$, $\mathbf{s} = (7\mathbf{i}) + 14\frac{5}{8}\mathbf{j}$	M1
	$SB = 2\frac{1}{8}$ (km) 2.125, 2.13	A1 **(4)**
		[13]
	OR When $t = 2\frac{3}{4}$, $7.5t - 18.5 = 2.125, 2.13$	M1 A1

Question 6(a)

First M1 for $\arctan\left(\frac{\pm 7.5}{\pm 12}\right)$ either way up
First A1 for a correct value from their expression, usually 32° or 58°
Second A1 for 302 (allow more accurate answers)

Question 6(b)

M1 for a clear attempt at $(40\mathbf{i} - 6\mathbf{j}) + t(-12\mathbf{i} + 7.5\mathbf{j})$
A1 for any correct expression

Question 6(c)

First M1 is really B1 for $4\mathbf{i} + 16.5\mathbf{j}$ (seen or implied but can be in unsimplified form)
Second M1 is for a subtraction, $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$.
Third DM1, dependent on second M1, for finding magnitude of their $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$
A1 for 5

Question 6(d)

First M1 for equating **i**-component of their answer in part (b) to 7 or
the **i**-component of their $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$ to zero

First A1 for 2.75 cao
Second M1 (independent) for attempt to find **j**-component of their **s** at their $t = 2.75$
Second A1 2.125 or 2.13 cao

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8.

Question Number	Scheme	Marks
7 (a)	$\sqrt{\left((-4)^2+8^2\right)}=\sqrt{80}\quad\left(\text{km h}^{-1}\right)$ accept exact equivalents or 8.9 or better	M1 A1 (2)
(b)	$\mathbf{p}=(2\mathbf{i}-8\mathbf{j})+t(-4\mathbf{i}+8\mathbf{j})$	B1 (1)
(c)	Equating **j** components $-8+8t=12-8t$ $t=\frac{5}{4}$ oe	M1 A1 A1 (3)
(d)	Using their t from (c) to find the **i**-cpts of **p** and **q and subtract them** $10\frac{1}{2}-(-3)=13\frac{1}{2}\quad(\text{km})$	M1 A1 ft A1 (3) **9**

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9.

Question Number	Scheme	Marks
7. **(a)**	$\tan\theta=\frac{3}{4}$; bearing is 37° **(nearest degree)**	M1; A1 (2)
(b)		
(i)	$\mathbf{p}=(\mathbf{i}+\mathbf{j})+t(2\mathbf{i}-3\mathbf{j})$	M1 A1
(ii)	$\mathbf{q}=(-2\mathbf{j})+t(3\mathbf{i}+4\mathbf{j})$	A1
(iii)	$\mathbf{PQ}=\mathbf{q}-\mathbf{p}=(-\mathbf{i}-3\mathbf{j})+t(\mathbf{i}+7\mathbf{j})$	M1 A1 (5)
(c)		
(i)	$-1+t=0$ $t=1$ or 3pm	M1 A1
(ii)	$-1+t=-(-3+7t)$ $t=\frac{1}{2}$ or 2.30 pm	M1 A1 (4) **11**

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10.

Question Number	Scheme	Marks
4. **(a)**	$\text{speed} = \sqrt{2^2 + (-5)^2}$ $= \sqrt{29} = 5.4$ or better	M1 A1 (2)
(b)	$((7\mathbf{i} + 10\mathbf{j}) - (2\mathbf{i} - 5\mathbf{j}))/5$ $= (5\mathbf{i} + 15\mathbf{j})/5 = \mathbf{i} + 3\mathbf{j}$ $\mathbf{F} = m\mathbf{a} = 2(\mathbf{i} + 3\mathbf{j}) = 2\mathbf{i} + 6\mathbf{j}$	M1 A1 A1 DM1 A1ft (5)
(c)	$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 5\mathbf{j}) + (\mathbf{i} + 3\mathbf{j})t$ $(-5 + 3t)\mathbf{j}$ Parallel to $\mathbf{i} \Rightarrow -5 + 3t = 0$ $t = 5/3$	M1 A1 M1 A1 (4) **[11]**

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11.

Question Number	Scheme	Marks
Q1	$(-4\mathbf{i} - 7\mathbf{j}) = \mathbf{r} + 4(-3\mathbf{i} + 2\mathbf{j})$ $\mathbf{r} = (8\mathbf{i} - 15\mathbf{j})$ $\lvert\mathbf{r}\rvert = \sqrt{8^2 + (-15)^2} = 17$ m	M1 A1 A1 M1 A1 ft **[5]**

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12.

Question Number	Scheme	Marks
Q7.	(a) $\mathbf{v} = \frac{21\mathbf{i} + 10\mathbf{j} - (9\mathbf{i} - 6\mathbf{j})}{4} = 3\mathbf{i} + 4\mathbf{j}$	M1 A1
	speed is $\sqrt{(3^2 + 4^2)} = 5\,(\text{km h}^{-1})$	M1 A1 (4)
	(b) $\tan\theta = \frac{3}{4}$ $(\Rightarrow \theta \approx 36.9°)$	M1
	bearing is 37, 36.9, 36.87, …	A1 (2)
	(c) $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j} + t(3\mathbf{i} + 4\mathbf{j})$	M1
	$= (3t + 9)\mathbf{i} + (4t - 6)\mathbf{j}$ * cso	A1 (2)
	(d) Position vector of S relative to L is	
	$(3T + 9)\mathbf{i} + (4T - 6)\mathbf{j} - (18\mathbf{i} + 6\mathbf{j}) = (3T - 9)\mathbf{i} + (4T - 12)\mathbf{j}$	M1 A1
	$(3T - 9)^2 + (4T - 12)^2 = 100$	M1
	$25T^2 - 150T + 125 = 0$ or equivalent	DM1 A1
	$(T^2 - 6T + 5 = 0)$	
	$T = 1, 5$	A1 (6)
		[14]